The commuting probability for the Sylow subgroups of finite and profinite groups

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Abstract. The commuting probability $\Pr(G, G)$ of a finite group G is the probability that two randomly chosen elements of a group G commute. The knowledge of $\Pr(G, G)$ gives information on the structure of G. For instance, it is known that if $\Pr(G, G) > 5/8$ then G is abelian, if $\Pr(G, G) > 5/8$ then G is nilpotent, if $\Pr(G, G) > 1/12$ then G is soluble. If X and Y are subsets of a finite group G, we can define the commuting probability $\Pr(X, Y)$ of X and Y. We will discuss how the values of commuting probability of suitable Sylow subgroups of a finite group G affect the structure of G. Some similar interesting questions can be asked also in the profinite setting.