Link between Strongly Regular Graphs and Divisible Design Graphs

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Abstract

In [1] Haemers and Higman studed a strongly regular graphs with a decomposition of their vertex sets in two parts such that the induced subgraphs on the parts are strongly regular, a clique, or a coclique. In [2, 3] a prolific construction of strongly regular graphs was found: it is based on a regular decomposition of a new divisible design graph (see [4]) with certain parameters and a clique, or a coclique. In [5], we determined the parameters of strongly regular graphs which admit a decomposition into a divisible design graph with any parameters and a coclique. In particular, it was shown that when the least eigenvalue of such a strongly regular graph is a prime power, its parameters coincide with those of the complement of a symplectic graph. As a counterpart of this result, we determine the parameters of all strongly regular graphs that can be decomposed into a divisible design graph and a Delsarte clique. In particular, an infinite family of strongly regular graphs with the required decomposition and a new infinite family of divisible design graphs are found.

References

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