Detecting properties of a finite group through the study of some functions on element orders

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Let G be a finite group. We denote by $\psi(G)$ the sum of element orders of G and by $o(G)$ the average order of G, that is

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element $x \in G$,

$$o(G) = \frac{\psi(G)}{|G|}.$$

Starting from a result due to H. Amiri, S.M. Jafarian Amiri and I.M. Isaacs on the function $\psi(G)$, in the last years there has been a growing interest in studying properties of these functions and their relations with the structure of G.

Properties of the group G when $\psi(G)$ satisfies some bounds have also been investigated.

Our aim in this talk is to report some results concerning the function $\psi(G)$ and some new results on the function $o(G)$.

In the last part of the talk some other functions related to the element orders will be considered.

References

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